MSc 17-12-2020

Th 1. Let V be a vector space over a field K.

(i) For any scalar $k \in K$ and $0 \in V$, $k0 = 0$.

(ii) For $0 \in K$ and any vector $u \in V$, $0u = 0$

(iii) If $ku = 0$, where $k \in K$ and $u \in V$ then $k = 0$ or $u = 0$

(iv) For any $k \in K$ and any $u \in V$,

$(-k)u = k(-u) = -ku$.

Examples of vector spaces.

Space K^n

Let K be an arbitrary field. The notation K^n is frequently used to denote the set of all n-tuples of elements in K. Here K^n is a vector space over K using the following operations.

(1) Vector addition $(a_1, a_2, \ldots a_n) + (b_1, b_2, \ldots b_n) = (a_1+b_1, a_2+b_2 \ldots a_n+b_n)$

Scalar Multiplication

$$K(a_1, a_2, \text{---} a_n) = (Ka_1, Ka_2 \text{--} Ka_n)$$

The zero vector in K^n is the n-tuple of zeros.

$$0 = (0, 0 \text{---} 0)$$

and the negative of a vector is defined by

$$-(a_1, a_2, \text{---} a_n) = (-a_1, -a_2, -a_3 \text{---} -a_n)$$

Polynomial space $P(t)$

Let $P(t)$ denote the set of all polynomial of the form

$$P(t) = a_0 + a_1(t) + a_2 t^2 + \text{---} + a_s t^s$$

$$(s = 1, 2, \text{---})$$

where the coefficient a_i belong to a field K. Then $P(t)$ is a vector space over K using the following operation

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(i) Vector addition – Here $P(t) + q(t)$ in $P(t)$ is the usual operation of addition of Polynomial.

(ii) <u>Scalar Multiplication</u> Here $Kp(t)$

in $P(t)$ is the usual operation of the product of a scalar k and a polynomial $p(t)$

The zero polynomial 0 is the zero vector in $P(t)$.